Modelling diameter distribution of natural Teak stands with 3-parameter Weibull function.

Joshua Ogunyemi and Eddie Bevilacqua

State University of New York College of Environmental Science and Forestry (SUNY ESF)

ESF
State University of New York College of Environmental Science and Forestry

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Introduction

- Sustainable management of forest resources relies heavily on accurate modeling of their structural attributes (Sa et al., 2023), particularly diameter distribution (Pogoda et al., 2019; Sa et al., 2023), which uses a probability density function (PDF) to distribute stand attributes across size classes such as the diameter at breast height (DBH) or tree height (Zhang et al., 2003).

- Probability density functions (PDF) are continuous functions that define the likelihood of a random variable taking a particular value, such that you can find the probability of the value using the function.

Introduction

- In Nigeria, where natural and planted teak (*Tectona grandis*) stands are a fast-growing tropical hardwood tree species and represent a significant forest resource, the accurate characterization of diameter distribution is crucial for sustainable forest management practices.

Objectives

- To assess the effectiveness of a three-parameter Weibull distribution in capturing the variability and patterns inherent in natural teak forest diameter distributions using both maximum likelihood (MLE) and moment-based (MoM) estimation methods.

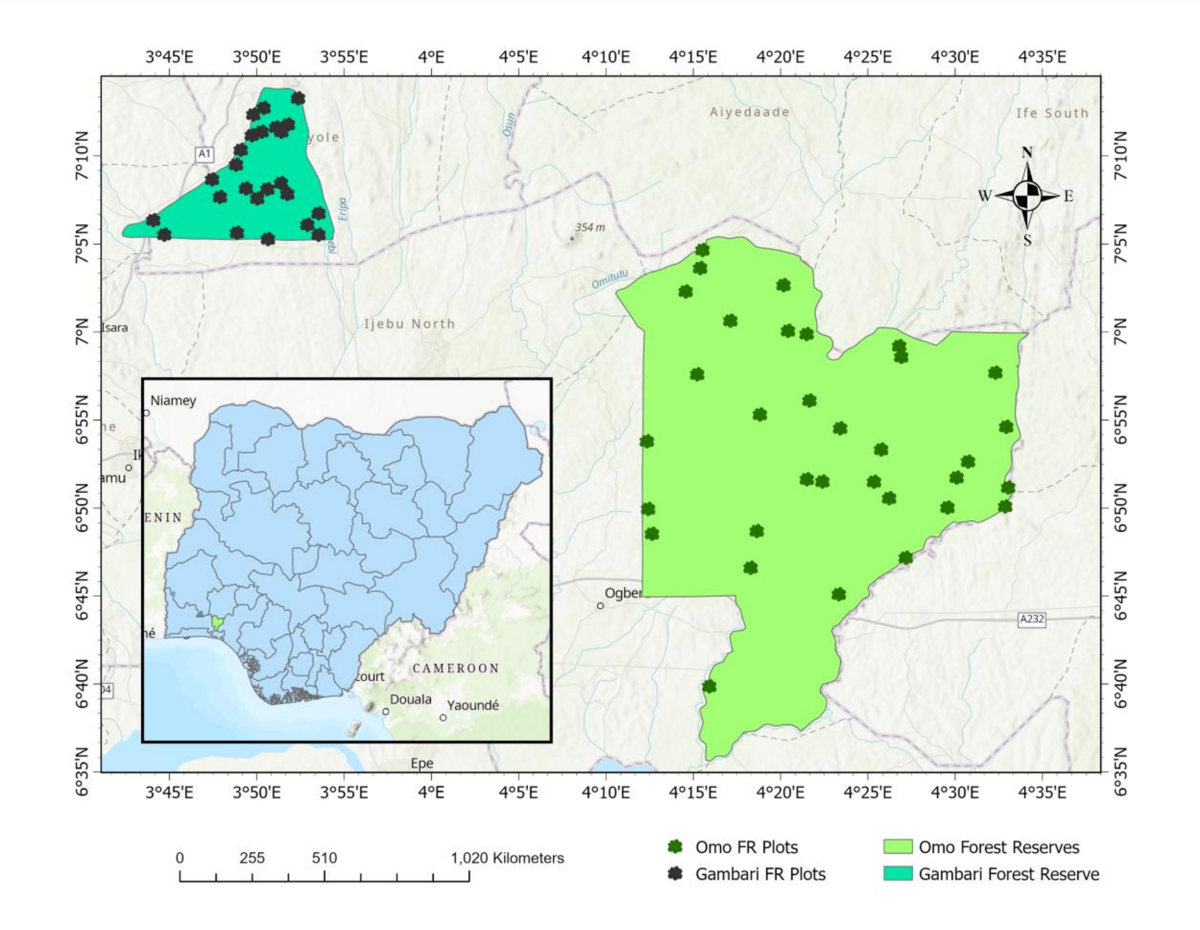
- To model the shape and scale parameters using the two-step parameter prediction method where we first fit the distribution to a set of plots or stands and then use the results to develop prediction models for the parameters from stand characteristics.

Materials and Methods

Study Area and Data Description

The data used for this study were collected from two forest reserves located in Nigeria:

- Omo Forest Reserve
- Gambari Forest Reserve.
- Fifty-seven (57) sample plots of size 25 m x 25 m in natural teak stands from the two forest reserves were randomly selected and positioned in the study area.



- Within each plot, height were measured using a clinometer (precision of 1m) and diameter-at-breast height (1.3 m above the ground) using a diameter tape (precision of 0.1 cm) were measured on all trees.

Fitting Diameter Distribution

- The **Three-parameter Weibull function** was used to describe the diameter distributions. The probability density function of the three-parameter Weibull distribution is (Bailey and Dell, 1973) (Eq. 1):

$$\boldsymbol{f(x) = \frac{\beta}{\eta}\left(\frac{x-\alpha}{\eta}\right)^{\beta-1} e^{\left[-\left(\frac{x-\alpha}{\eta}\right)^{\beta}\right]}} \qquad [1]$$

- Where x is diameter at breast height, β is the shape parameter ($\beta > 0$), η is the scale parameter ($\eta > 0$); and α is the location parameter.

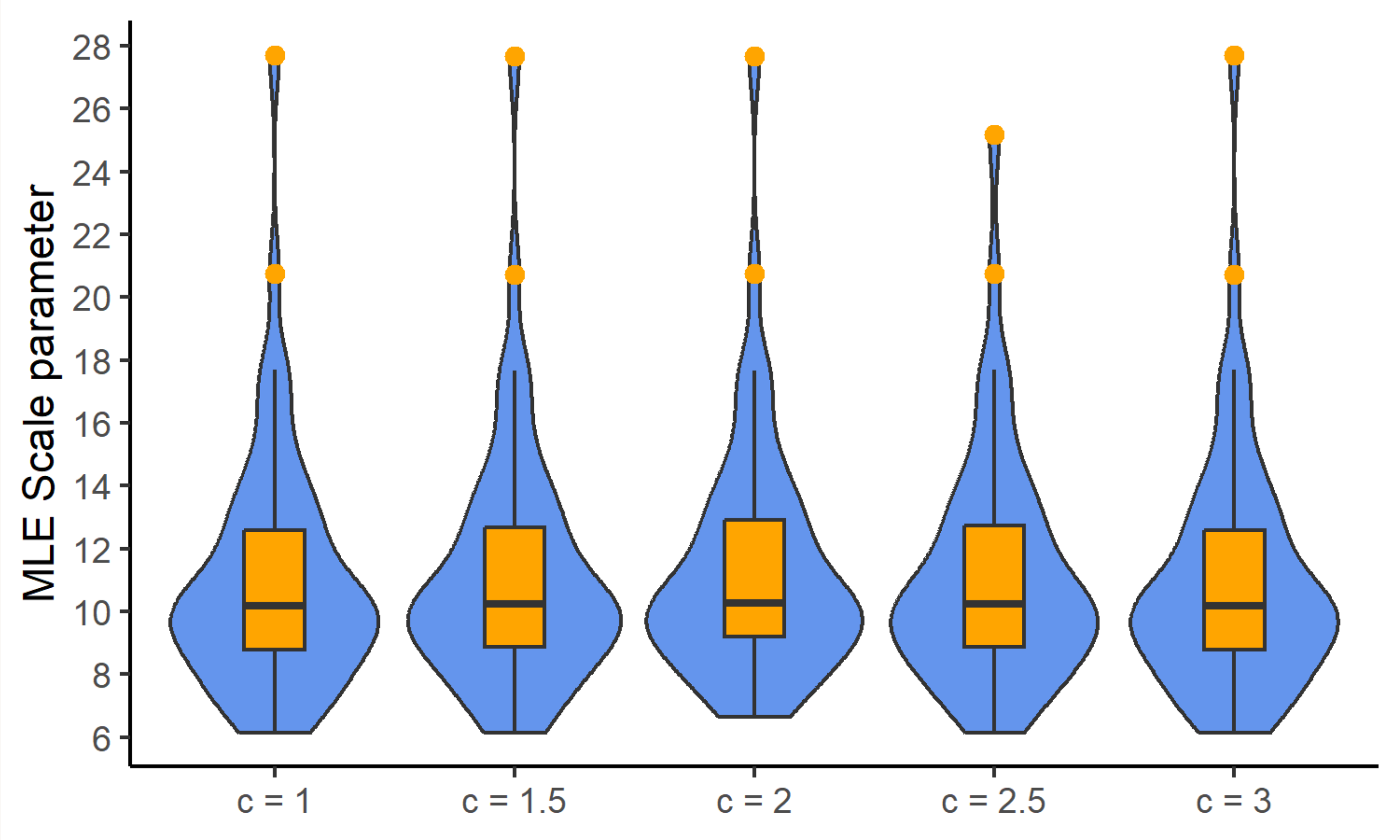
Fitting Diameter Distribution

- The initial parameter estimate used for location was (D_m – c), where D_m represents the minimum diameter and different values (1.0, 1.5, 2.0, 2.5 and 3.0) were tried to choose a reasonable value for c.

```
### Specify the function to fit ####
dW3 <- function(x, a, b, c){
  (c/b) * ((x-a)/b)^(c-1) * exp(-((x-a)/b)^c)}
# the pdf of 3P Weibull (a: location, b:scale, c:shape/slope)
## Where:
## f(t) >= 0; random variable = x

```

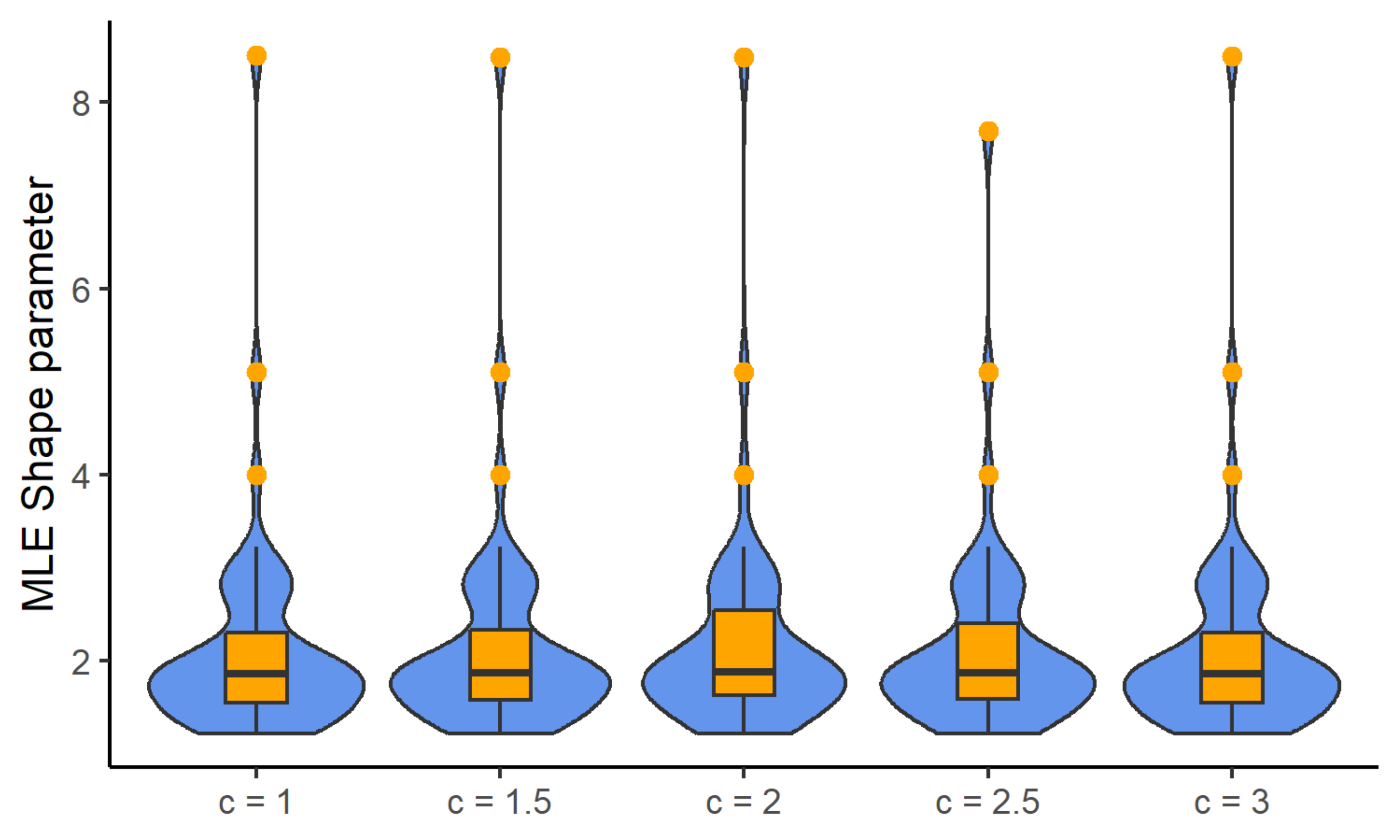
```
##### Use of loops ########################################
f1 <- function(plotX){
  fitW3 <- mledist(plotX, "W3", start=list(a=min(plotX)-3,b=as.numeric(quantile(plotX, 0.632)),c=3))
  return(fitW3$estimate)
}

## In a 3-parameter Weibull distribution, the scale parameter is directly related to the 63.2 percentile, meaning it represents the value at which 63.2% of the data falls below in the distribution.

```

Variability in parameter at changing values of c



Scale (η)

Variability in parameter at changing values of c



Shape (β)

Modelling Distribution Parameters

- Quadratic mean diameter (Qmd), basal area (BA), number of stems per hectare (Tph), and Lorey's mean height (Lht, used as a measure of site productivity) were used as explanatory variables in the prediction models.

- No data was collected on age.

- Using linear regression, prediction models for the Weibull parameters [shape (β) and scale (η)] were developed.

Descriptive statistics of the predictor variables

Site	Variable	N	Mean	Min	Max	S.D
Omo	Tph	33	870.30	624	1744	191.83
	Qmd (cm)	33	18.48	14.28	22.38	2.29
	BA (m^2/ha)	33	23.93	11.54	50.78	8.81
	Lht (m)	33	17.36	14.65	20.37	1.56
Gambari	Tph	24	597.33	480	864	101.38
	Qmd (cm)	24	20.57	16.21	23.55	1.81
	BA (m^2/ha)	24	20.01	12.88	28.39	4.78
	Lht (m)	24	19.04	15.90	23.22	1.69

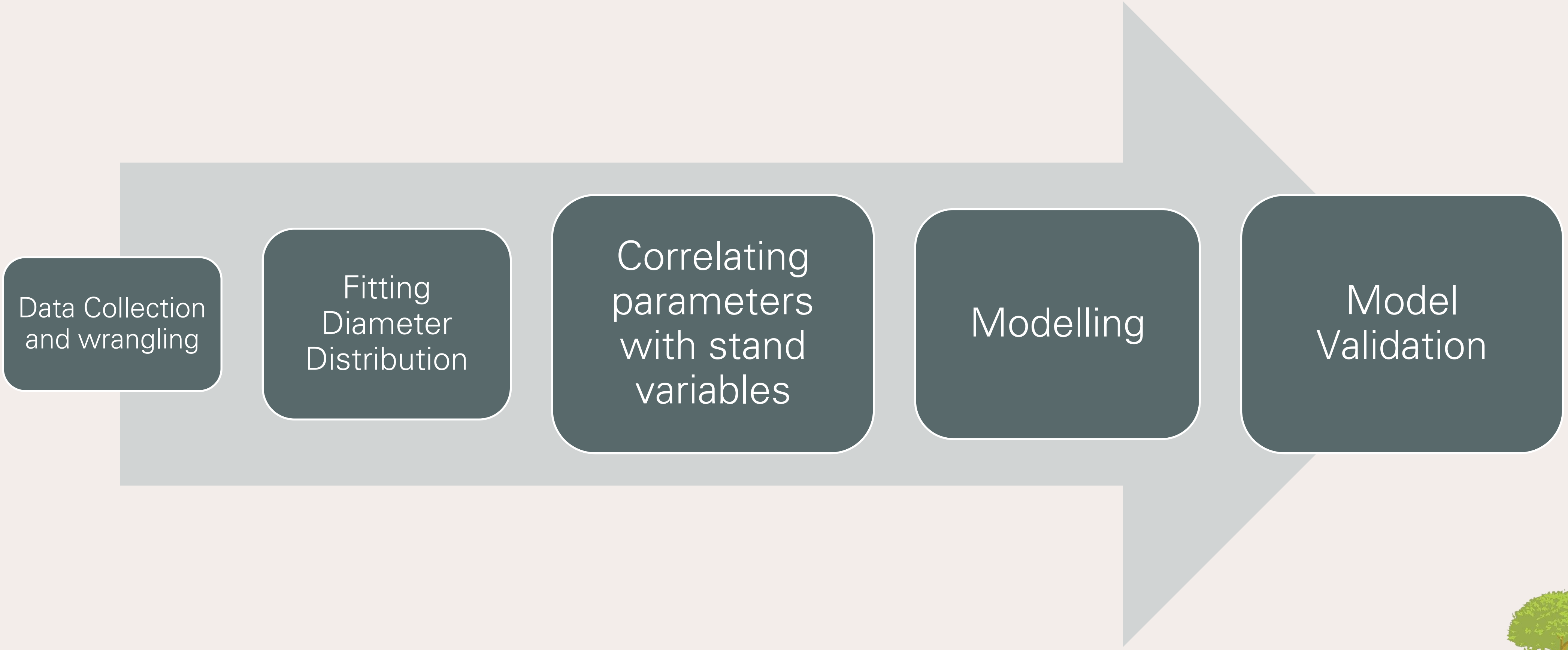
Model Validation

- The leave-one-out cross-validation (LOOCV) method was used in the validation of each of the regression models developed for the shape (β) and scale (η) parameters of the three-parameter Weibull function.

- This validation method is a widely-used resampling technique in model validation and was selected for this study because of its suitability for validating our regression models that were trained with a small dataset of 50 plots.

- In each iteration, one data point is held out as the test set, and the model is trained on the remaining data points. This process is repeated for each observation, ensuring that every data point is used once as a validation sample. At the end, a diagnostic check on the residuals.

Workflow

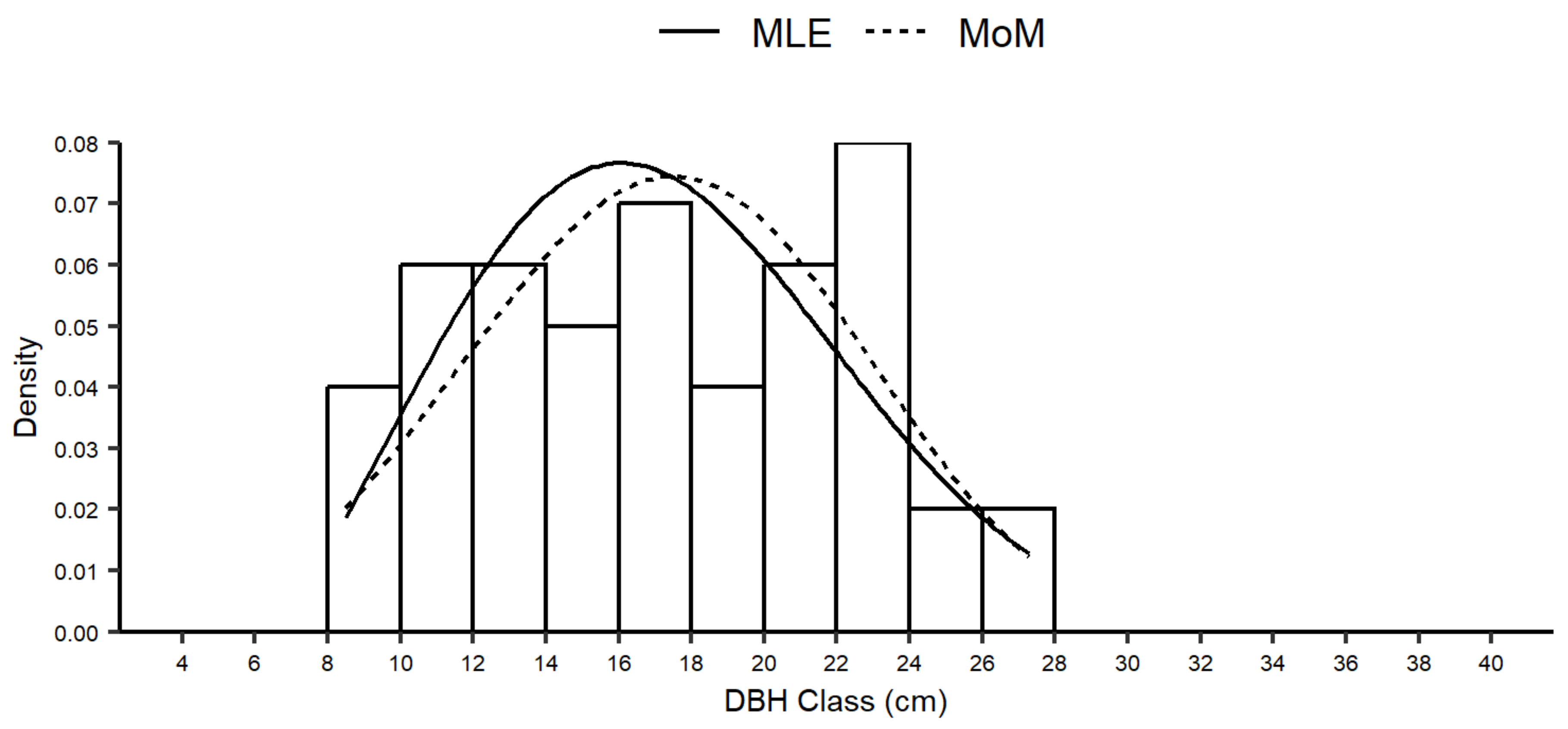

Data Collection and wrangling
Fitting Diameter Distribution
Correlating parameters with stand variables
Modelling
Model Validation

Results & Discussion

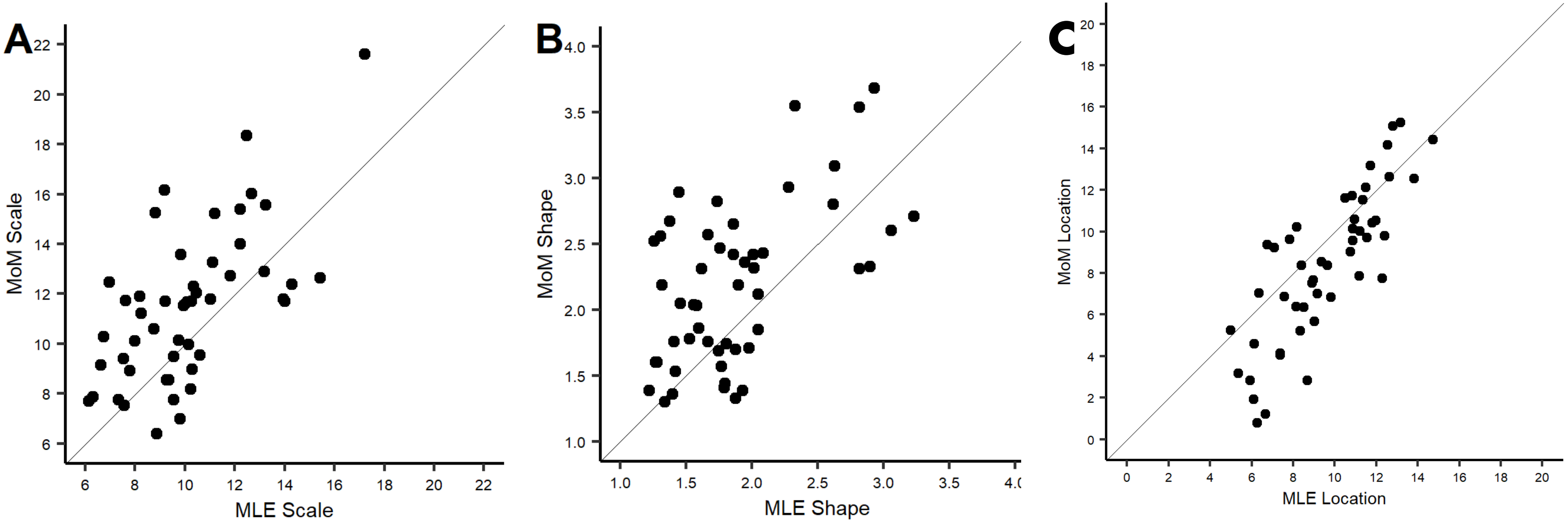
Summary Statistics of Weibull estimates for both MLE and MoM

Parameter	Sites	Method	Mean	Min	Max	S.D
Scale (η)	Omo	MLE	10.38	6.14	27.71	4.39
		MoM	11.96	7.70	22.80	4.07
	Gambari	MLE	12.04	7.56	17.69	2.72
		MoM	13.54	6.38	23.06	4.18
Shape (β)	Omo	MLE	2.09	1.22	8.50	1.36
		MoM	2.41	1.39	6.66	1.09
	Gambari	MLE	2.20	1.34	4.00	0.72
		MoM	2.35	1.30	4.41	0.78

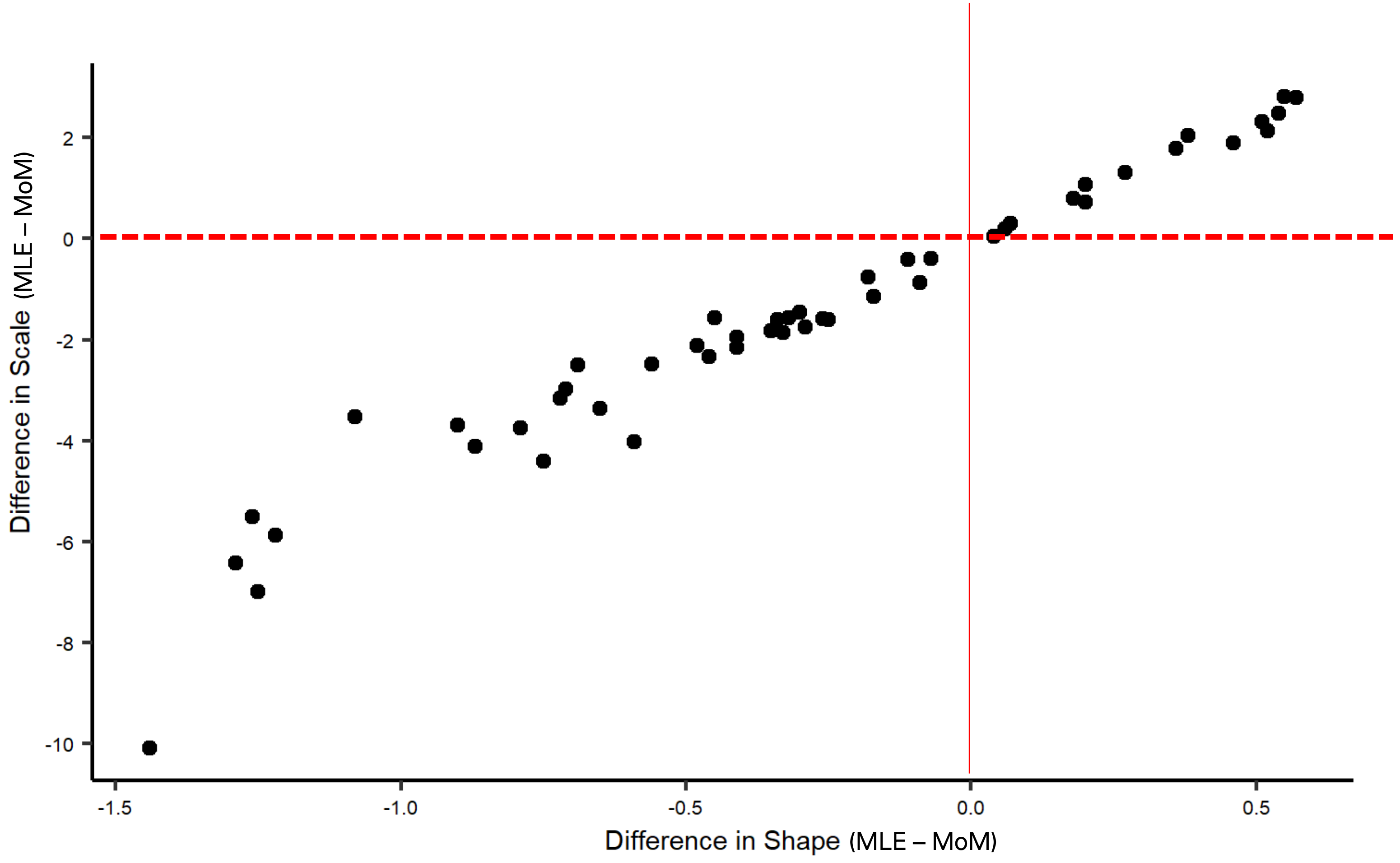
MLE and MoM fit over DBH histogram



3-Parameter Weibull estimates



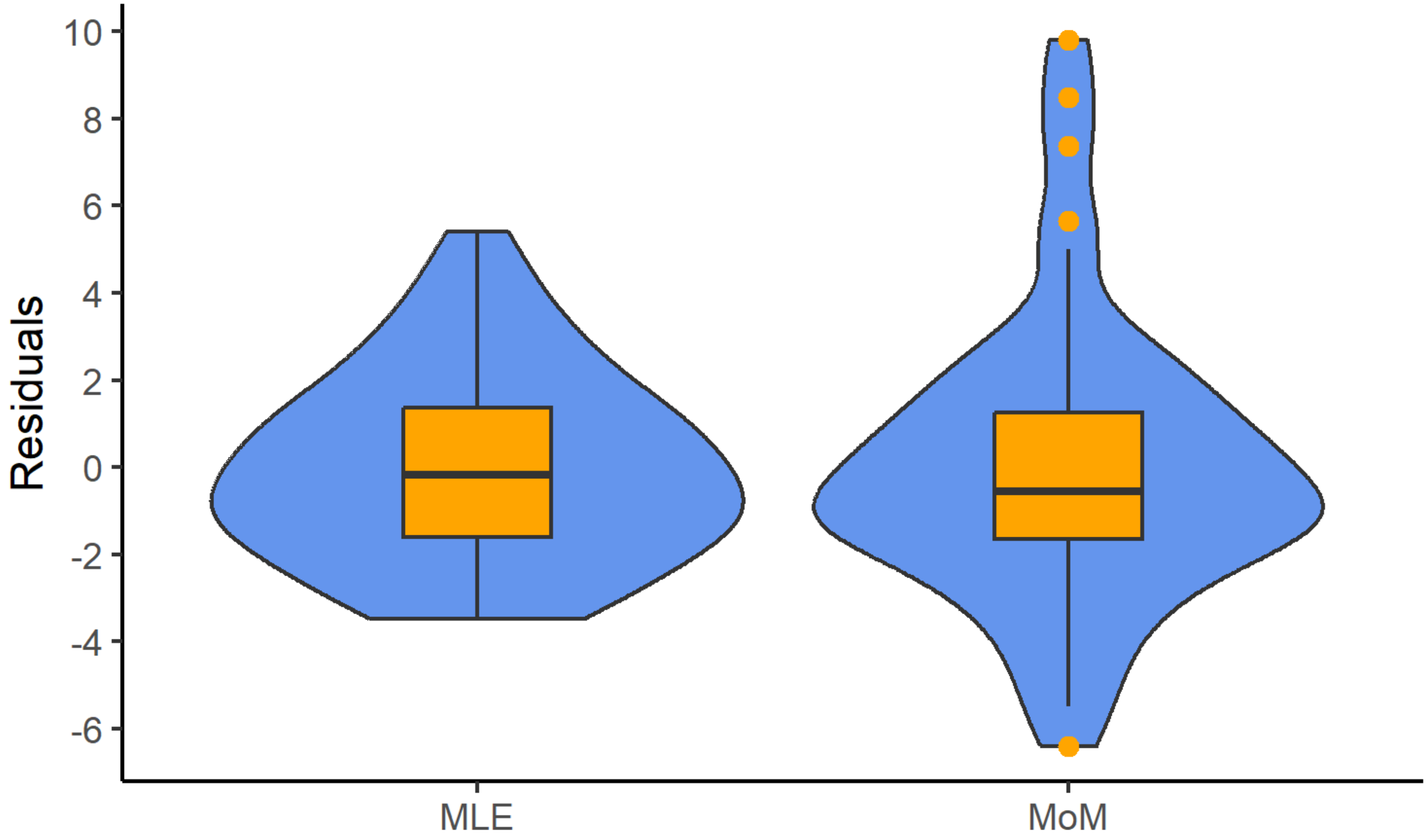
Joint Distribution of parameters



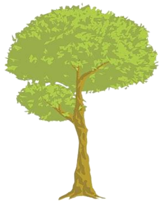
Regression coefficients and fit indices of models for Weibull parameters

		Predictor variables					Goodness of fit	
Model	Method	Intercept	Tph	Qmd	Ba	Lht	R^2	RMSE
Scale (η)	MLE	6.788	-0.005	0.363	-	-	0.299	2.107
	MoM	18.824	-0.010	-	-	-	0.176	3.182
Shape (β)	MLE	5.613	-0.813	-	0.535	-	0.108	0.499
	MoM	2.929	-	-	-0.034	-	0.102	0.581

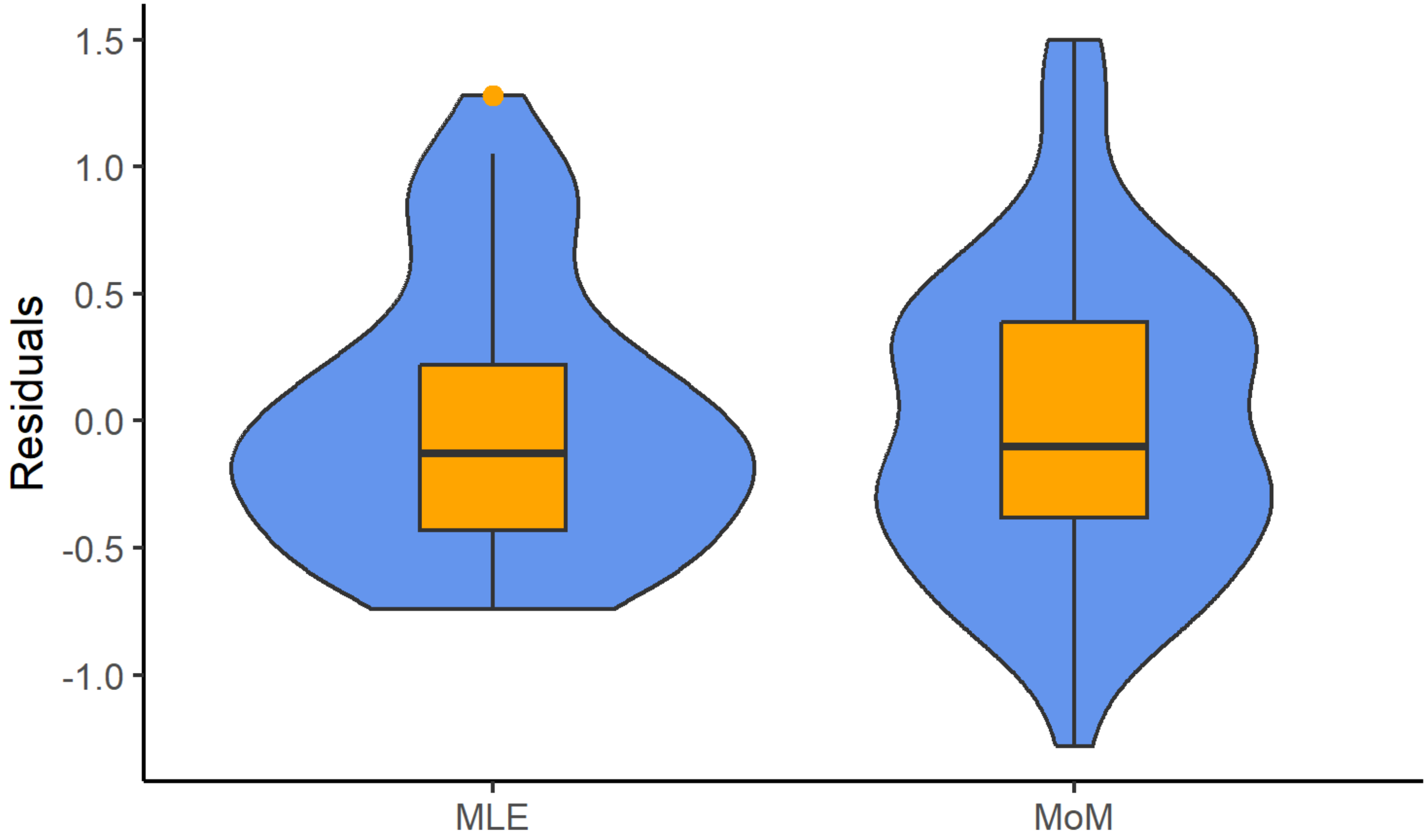
Residuals plot from LOOCV



Scale (η)



Residuals plot from LOOCV



Shape (β)

Conclusion

Conclusion

- This study demonstrates the effectiveness of the Method of Moments (MoM) over Maximum Likelihood Estimation (MLE) in estimating Weibull distribution parameters for natural teak forests in Omo and Gambari Forest Reserves.

- Although, we did not have as much variability as expected in the parameters, Quadratic mean diameter, Trees per hectare, and Basal Area were significant stand variables that could potentially be used explaining the variation in Weibull parameters.

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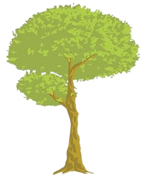

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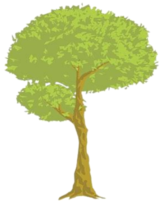

Thank you

Olakunle Joshua

315-952-8812

oogunyemi@esf.edu